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Mark Scheme (Results)
January 2008

GCE

GCE Mathematics (6664/01)

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January 2008
6664 Core Mathematics C2
Mark Scheme

Question Number	Scheme	Marks
1. a)i) ii)	f(3) = 3^3 - 2 x 3^2 - 4 x 3 + 8 ; = 5 f(–2) = (– 8 – 8 + 8 + 8) = 0 (B1 on Epen, but A1 in fact) M1 is for attempt at either f(3) or f(–3) in (i) or f(–2) or f(2) in (ii).	M1; A1 A1 (3)
(b)	$[(x + 2)](x^2 - 4x + 4)$ (= 0 not required) [must be seen or used in (b)] $(x + 2)(x - 2)^2$ (= 0) (can imply previous 2 marks) Solutions: $x = 2$ or -2 (both) or (–2, 2, 2) A1 (4)	M1 A1 M1 [7]
Notes: (a)	No working seen: Both answers correct scores full marks One correct ;M1 then A1B0 or A0B1, whichever appropriate. <u>Alternative (Long division)</u> Divide by (x – 3) OR (x + 2) to get $x^2 + ax + b$, *a* may be zero [M1] $x^2 + x - 1$ and $+5$ seen i.s.w. (or "remainder = 5") [A1] $x^2 - 4x + 4$ and 0 seen (or "no remainder") [B1]	
(b)	First M1 requires division by a found factor ; e.g $(x + 2)$, $(x - 2)$ or what candidate thinks is a factor to get $(x^2 + ax + b)$, a may be zero. First A1 for $[(x + 2)](x^2 - 4x + 4)$ or $(x - 2)(x^2 - 4)$ Second M1:attempt to factorise their found quadratic. (or use formula correctly) [Usual rule: $x^2 + ax + b = (x + c)(x + d)$, $where \mid cd \mid = \mid b \mid$.] **N.B.** Second A1 is for solutions, not factors *<u>Alternative (first two marks)</u>* $(x+2)(x^2 + bx + c) = x^3 + (2+b)x^2 + (2b+c)x + 2c = 0$ and then compare with $x^3 - 2x^2 - 4x + 8 = 0$ to find *b* and *c*. [M1] $b = -4$, $c = 4$ [A1] *<u>Method of grouping</u>* $x^3 - 2x^2 - 4x + 8 = x^2(x-2)$,$4(x \pm 2)$ M1; = $x^2(x-2) - 4(x-2)$ A1 [= $(x^2 - 4)(x - 2)$] = $(x+2)(x-2)^2$ M1 Solutions: $x = 2$, $x = -2$ both A1	
2. (a)	Complete method, using terms of form ar^k, to find r [e.g. **Dividing** ar^6 = 80 by ar^3 = 10 to find r ; $r^6 - r^3$ = 8 is M0] r = 2	M1 A1 (2)
(b)	Complete method for finding a [e.g. Substituting value for *r* into equation of form ar^k = 10 or 80 and finding a value for *a*.]	M1

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	($8a$ = 10) $a = \frac{5}{4} = 1\frac{1}{4}$ (equivalent single fraction or 1.25)	A1 (2)
(c)	Substituting their values of a and r into **correct** formula for sum. $S = \frac{a(r^n - 1)}{r-1} = \frac{5}{4}(2^{20} - 1)$ (= 1310718.75) 1 310 719 (only this)	M1 A1 (2) **[6]**
Notes:	(a) M1: Condone errors in powers, e.g. ar^4 = 10 and/or ar^7 = 80, A1: For r = 2, allow even if ar^4 = 10 and ar^7 = 80 used (just these) (M mark can be implied from numerical work, if used correctly) (b) M1: Allow for numerical approach: e.g. $\frac{10}{r_c^{\,3}} \leftarrow \frac{10}{r_c^{\,2}} \leftarrow \frac{10}{r_c} \leftarrow 10$ In (a) and (b) correct answer, with no working, allow both marks. (c) Attempt 20 terms of series and add is M1 (correct last term 655360) If formula **not** quoted, errors in applying their a and/or r is M0 Allow full marks for correct answer with no working seen.	
3. (a)	$\left(1+\frac{1}{2}x\right)^{10}$ **=** 1 + $\underline{\binom{10}{1}\left(\frac{1}{2}x\right)+\binom{10}{2}\left(\frac{1}{2}x\right)^2+\binom{10}{3}\left(\frac{1}{2}x\right)^3}$ = 1 + 5x ; + $\frac{45}{4}$ (or 11.25)x^2 + $15x^3$ (coeffs need to be these, i.e, simplified) [Allow A1A0, if totally correct with unsimplified, single fraction coefficients)	M1 A1 A1; A1 (4)
(b)	(1 + $\frac{1}{2}\times 0.01$)^{10} = 1 + 5(0.01) + ($\frac{45}{4}$ $or 11.25)(0.01)^2$ + 15(0.01)^3 = 1 + 0.05 + 0.001125 + 0.000015 = 1.05114 cao	M1 A1√ A1 (3) **[7]**
Notes:	(a) For M1 first A1: Consider underlined expression only. M1 Requires correct structure for at least two of the three terms: (i) Must be attempt at binomial coefficients. (ii) Must have increasing powers of x , (iii) May be listed, need not be added; *this applies for all marks*. First A1: Requires all three correct terms but need not be simplified, allow 1^{10} etc, ${}^{10}C_2$ etc, and condone omission of brackets around powers of ½x Second A1: Consider as B1 for 1 + 5x (b) For M1: Substituting their (0.01) into their (a) result First A1 (f.t.): Substitution of (0.01) into their 4 termed expression in (a) Answer with no working scores no marks (calculator gives this answer)	

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4. (a)	$3\sin^2\theta - 2\cos^2\theta = 1$ $3\sin^2\theta - 2(1 - \sin^2\theta) = 1$ (M1: Use of $\sin^2\theta + \cos^2\theta = 1$) $3\sin^2\theta - 2 + 2\sin^2\theta = 1$ $5\sin^2\theta = 3$ cso AG	M1 A1	(2)
(b)	$\sin^2\theta = \frac{3}{5}$, so $\sin\theta = (\pm)\sqrt{0.6}$ Attempt to solve both $\sin\theta = +$.. and $\sin\theta = -$ (may be implied by later work) M1 $\theta = 50.7685°$ awrt $\theta = 50.8°$ (dependent on first M1 only) θ ($= 180° - 50.7685_c°$); $= 129.23...°$ awrt $129.2°$ [f.t. dependent on first M and 3rd M] $\sin\theta = -\sqrt{0.6}$ $\theta = 230.785°$ and $309.23152°$ awrt $230.8°$, $309.2°$ (both)	M1 A1 M1; A1 √ M1A1	 (7) [9]
Notes:	(a) N.B: AG; need to see at least one line of working after substituting $\cos^2\theta$. (b) First M1: Using $5\sin^2\theta = 3$ to find value for $\sin\theta$ or θ Second M1: Considering the – value for $\sin\theta$. (usually later) First A1: Given for awrt 50.8°. Not dependent on second M. Third M1: For $(180 - 50.8_c)°$, need not see written down Final M1: Dependent on second M (but may be implied by answers) For (180 + candidate's 50.8)° or $(360 - 50.8_c)°$ or equiv. Final A1: Requires both values. (no follow through) [Finds $\cos^2\theta = k$ (k = 2/5) and so $\cos\theta = (\pm)$...M1, then mark equivalently]		

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5.	<u>Method 1</u> (Substituting a = 3b into second equation at some stage)	
	Using a law of logs correctly (anywhere) e.g. $\log_3 ab = 2$	M1
	Substitution of 3*b* for *a* (or a/3 for b) e.g. $\log_3 3b^2 = 2$	M1
	Using base correctly on correctly derived $\log_3$ p= q e.g. $3b^2 = 3^2$	M1
	First correct value $b = \sqrt{3}$ (allow $3^{1/2}$)	A1
	Correct method to find other value (dep. on at least first M mark)	M1
	Second answer $a = 3b = 3\sqrt{3}$ or $\sqrt{27}$	A1
	<u>Method 2</u> (Working with two equations in $\log_3 a$ and $\log_3 b$)	
	" Taking logs" of first equation and " separating" $\log_3 a = \log_3 3 + \log_3 b$ $(= 1 + \log_3 b)$	M1
	Solving simultaneous equations to find $\log_3 a$ or $\log_3 b$ [$\log_3 a = 1\frac{1}{2}$, $\log_3 b = \frac{1}{2}$]	M1
	Using base correctly to find a or b	M1
	Correct value for *a* or *b* $a = 3\sqrt{3}$ or $b = \sqrt{3}$	A1
	Correct method for second answer, dep. on first M; correct second answer [Ignore negative values]	M1;A1**[6]**
Notes:	Answers must be exact; decimal answers lose both A marks There are several variations on Method 1, depending on the stage at which *a* = 3*b* is used, but they should all mark as in scheme. In this method, the first three method marks on Epen are for (i) First M1: correct use of log law, (ii) Second M1: substitution of *a* = 3*b*, (iii) Third M1: requires using base correctly on correctly derived $\log_3$ p= q	

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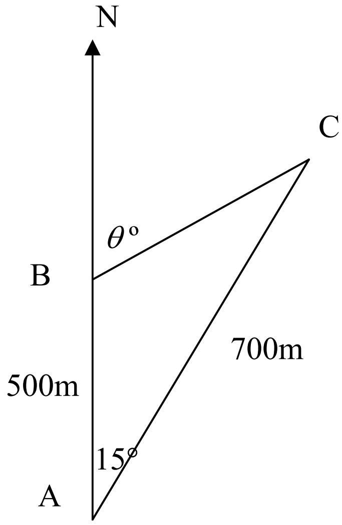
6.



(a)

$BC^2 = 700^2 + 500^2 - 2 \times 500 \times 700 \cos 15°$ — M1 A1

$(= \;\; 63851.92\ldots)$

$BC \;=\; 253$ awrt — A1 (3)

(b)

$\frac{\sin B}{700} = \frac{\sin 15}{\text{candidate's } BC}$ — M1

$\sin B = \sin 15 \times 700 / 253_c = 0.716..$ and giving an **obtuse** B (134.2°) dep — M1

$\theta = 180° -$ candidate's angle B (Dep. on first M only, B can be acute) — M1

$\theta = 180 - 134.2 = (0)45.8$ (allow 46 or awrt 45.7, 45.8, 45.9) — A1 (4) **[7]**

[46 needs to be from correct working]

Notes:

(a) If use cos 15° = ..., then A1 not scored until written as BC^2 = … correctly

Splitting into 2 triangles BAX and CAX, where X is foot of perp. from B to AC

Finding value for BX and CX and using Pythagoras — M1

$BC^2 = (500\sin 15°)^2 + (700 - 500\cos 15°)^2$ — A1

$BC \;=\; 253$ awrt — A1

(b) Several alternative methods: (Showing the M marks, 3rd M dep. on first M))

(i) $\cos B = \frac{500^2 + \text{candidate's}BC^2 - 700^2}{2\text{x}500\text{xcandidate's}BC}$ or $700^2 = 500^2 + BC_c{}^2 - 2x500xBC_c$ M1

Finding angle B M1, then M1 as above

(ii) 2 triangle approach, as defined in notes for (a)

$\tan CBX \;=\; \frac{700 - valueforAX}{valueforBX}$ — M1

Finding value for $\angle CBX$ $(\approx 59°)$ — M1

$\theta = [180° - (75° + candidate's\ \angle CBX)]$ — M1

(iii) Using sine rule (or cos rule) to find C first:

Correct use of sine or cos rule for C M1, Finding value for C M1

Either $B = 180° - (15° +$ candidate's $C)$ or $\theta = (15° +$ candidate's $C)$ M1

(iv) $700\cos 15° = 500 + BC\cos\theta$ M2 {first two Ms earned in this case}

Solving for θ; $\theta = 45.8$ (allow 46 or5.7, 45.8, 45.9 M1;A1

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7 (a)	Either solving $0 = x(6 - x)$ and showing $x = 6$ (and $x = 0$) or showing (6,0) (and $x = 0$) satisfies $y = 6x - x^2$ [allow for showing x = 6]	B1 (1)
(b)	Solving $2x = 6x - x^2$ $(x^2 = 4x)$ to x = .. $x = 4$ (and $x = 0$) Conclusion: when $x = 4$, $y = 8$ and when $x = 0$, $y = 0$,	M1 A1 A1 (3)
(c)	(Area =) $\int_{(0)}^{(4)}(6x - x^2)\,dx$ Limits not required Correct integration $3x^2 - \frac{x^3}{3}$ (+ c) Correct use of correct limits on their result above (see notes on limits) [" $3x^2 - \frac{x^3}{3}$ "]^4 − [" $3x^2 - \frac{x^3}{3}$ "]$_0$ with limits substituted $[= 48 - 21\frac{1}{3} = 26\frac{2}{3}]$ Area of triangle = $2 \times 8 = 16$ (Can be awarded even if no M scored, i.e. B1) Shaded area = $\pm$(area under curve − area of triangle) applied correctly $(= 26\frac{2}{3} - 16) = 10\frac{2}{3}$ (awrt 10.7)	M1 A1 M1 A1 M1 A1 (6)**[10]**
Notes	(b) In scheme first A1: need only give $x = 4$ If *verifying approach* used: Verifying (4,8) satisfies both the line and the curve M1(attempt at both), Both shown successfully A1 For final A1, (0,0) needs to be mentioned ; accept " clear from diagram" (c) **Alternative** Using Area = $\pm\int_{(0)}^{(4)}\{(6x - x^2);\ -2x\}\,dx$ approach (i) If candidate integrates separately can be marked as main scheme If combine to work with = $\pm\int_{(0)}^{(4)}(4x - x^2)\,dx$, **first** M mark and **third** M mark $= (\pm)\,[\,2x^2 - \frac{x^3}{3}\ (+ c)\,]$ A1, Correct use of correct limits on their result **second** M1, Totally correct, unsimplified ± expression (may be implied by correct ans.) A1 10⅔ A1 [Allow this if, having given - 10⅔, they correct it] M1 *for correct use of correct limits*: Must substitute correct limits for their strategy into a changed expression and subtract, either way round, e.g $\pm\{[\ ]^4 - [\ ]_0\}$ If a long method is used, e,g, finding three areas, this mark only gained for correct strategy and all limits need to be correct for this strategy. Use of trapezium rule: M0A0MA0,possibleA1for triangle M1(if correct application of trap. rule from $x = 0$ to $x = 4$) A0	

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8	(a)	$(x-6)^2+(y-4)^2=;\ 3^2$	B1; B1 (2)
	(b)	Complete method for MP: $=\sqrt{(12-6)^2+(6-4)^2}$	M1
		$=\sqrt{40}$ (= 6.325)	A1
		[These first two marks can be scored if seen as part of solution for (c)]	
		Complete method for $\cos\theta$, $\sin\theta$ or $\tan\theta$	M1
		e.g. $\cos\theta = \frac{\text{MT}}{\text{MP}} = \frac{3}{candidate's\sqrt{40}}$ (= 0.4743) (θ = 61.6835°)	
		[If TP = 6 is used, then M0]	
		$\theta = 1.0766$ rad **AG**	A1 (4)
	(c)	Complete method for area TMP; e.g. $=\frac{1}{2}\times 3\times\sqrt{40}\sin\theta$	M1
		$=\frac{3}{2}\sqrt{31}$ (= 8.3516..) allow awrt 8.35	A1
		Area (sector) MTQ $= 0.5\times 3^2\times 1.0766$ (= 4.8446…)	M1
		Area TPQ = candidate's (8.3516.. – 4.8446..)	M1
		= 3.507 awrt	A1 (5)
		[Note: 3.51 is A0]	**[11]**
Notes		(a) Allow 9 for 3^2. (b) First M1 can be implied by √ 40 For second M1: May find TP = $\sqrt{(\sqrt{40})^2-3^2}=\sqrt{31}$, then either $\sin\theta=\frac{TP}{MP}=\frac{\sqrt{31}}{\sqrt{40}}$ (= 0.8803...) *or* $\tan\theta=\frac{\sqrt{31}}{3}$ (1.8859..) or cos rule **NB. Answer is given, but allow final A1 if all previous work is correct.** (c) First M1: (alternative) $\frac{1}{2}\times 3\times\sqrt{40-9}$	

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9 (a)	(Total area) = $3xy + 2x^2$	B1
	(Vol:) $x^2y = 100$ $\left(y = \frac{100}{x^2}, xy = \frac{100}{x}\right)$	B1
	Deriving expression for area in terms of x only (Substitution, or clear use of, y or xy into expression for area)	M1
	(Area =) $\frac{300}{x} + 2x^2$ **AG**	A1 cso (4)
(b)	$\frac{dA}{dx} = -\frac{300}{x^2} + 4x$	M1A1
	Setting $\frac{dA}{dx} = 0$ and finding a value for correct power of x, for cand. M1 [$x^3 = 75$] $x = 4.2172$ awrt 4.22 (allow exact $\sqrt[3]{75}$)	A1 (4)
(c)	$\frac{d^2A}{dx^2} = \frac{600}{x^3} + 4$ = positive therefore minimum	M1A1 (2)
(d)	Substituting found value of x into (a) (Or finding y for found x and substituting both in $3xy + 2x^2$) [$y = \frac{100}{4.2172^2} = 5.6228$]	M1
	Area = 106.707 awrt 107	A1 (2) **[12]**
Notes	(a) First B1: Earned for correct unsimplified expression, isw. (c) For M1: Find $\frac{d^2A}{dx^2}$ and explicitly consider its sign, state > 0 or "positive" A1: Candidate's $\frac{d^2A}{dx^2}$ must be correct for their $\frac{dA}{dx}$, sign must be + ve and conclusion "so minimum", (allow QED, √). (may be wrong x, or even no value of x found) <u>Alternative:</u> M1: Find value of $\frac{dA}{dx}$ on either side of "$x = \sqrt[3]{75}$" and consider sign A1: Indicate sign change of negative to positive for $\frac{dA}{dx}$, and conclude minimum. OR M1: Consider values of A on either side of "$x = \sqrt[3]{75}$" and compare with"107" A1: Both values greater than "$x = 107$" and conclude minimum. Allow marks for (c) and (d) where seen; even if part labelling confused.	

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